

Strong coupling between two spin ensembles via a large Josephson junction

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We propose a method to achieve strong coupling between a spin ensemble and a large Josephson junctions (LJJ). Then, the strong coupling between two spin ensembles can be induced by a LJJ. A non-adiabatic holonomic single-qubit quantum gates is realized. Moreover, with the dispersive interaction between the spin ensembles and the LJJ, the high-fidelity two-qubit phase gate can be implemented with two spin ensembles within an operation time 0.41 ns and π phase-swap gate can be realized within an operation time 103.11 ns with a high fidelity greater than 99%.

PACS numbers: 03.67.Lx, 76.30.Mi

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I. INTRODUCTION

Due to the sufficiently long electronic spin lifetime as well as the possibility of coherent manipulation at room temperature [1], the nitrogen-vacancy (NV) center in diamond provides an arena to study various macroscopic quantum phenomena and acts as a perfect candidate toward quantum computers. Experimentally, with NV centers Deutsch-Jozsa quantum algorithm [2], quantum memory [3], quantum logical NOT and a conditional two-qubit gate [4], and decoherence-protected quantum gates for the electron-nuclear spin register [5], electron spin resonance detected by a superconducting qubit [6], and controlling spin relaxation [7] have been realized, respectively. Theoretically, a multi-qubit conditional phase gate with three NV centers coupled to a whispering-gallery mode cavity has been proposed [8], quantum-information transfer with NV centers coupled to a whispering-gallery microresonator has been proposed [9], and anomalous decoherence effect has been found in NV center [10].

Recently, the hybrid quantum system consisting of a flux qubit and a NV ensemble has been proposed [11]. The coupling strength between the flux qubit and the NV ensemble is stronger than that between NV centers and a transmission line resonator [11]. And, the strong coupling [12], quantum information transfer [13], as well as observation of dark states [14] have been demonstrated in this hybrid system. Also, many theoretical works have been done in the quantum information based on this hybrid system. For example, Refs. [15, 16] proposed the quantum information transfer between two spatially-separated NV ensembles. Hümmer *et al.* [17] proposed how to simulate a localization-delocalization transition based on an array of superconducting flux qubits which are coupled to a diamond crystal containing NV centers. Qiu *et al.* [18] showed that the coupling strength between flux qubits and NV ensembles can reach the strong and even ultrastrong coupling regimes by either engineering the hybrid structure in advance or tuning the excitation frequencies of NV ensembles via external magnetic fields. Lü *et al.* [19] proposed how to realize high-fidelity quantum storage using two coupled flux qubits and a NV ensemble. Song *et al.* [20] studied a scheme for creating macroscopic entangled coherent states of separate NV ensembles that are coupled to a flux qubit.

In this paper, we study a hybrid quantum system, which consists of the NV ensembles and a large Josephson junctions (LJJ), as shown in Fig.1. The key point of our scheme is that the strong coupling between the LJJ and the NV ensembles can be achieved. And a non-adiabatic holonomic single-qubit quantum gates is implemented. Also, in the large detuning regime between LJJ and NV ensembles, the LJJ can induce the strong interaction of two spatially-separated NV ensembles. Considering decoherence in experimentally available systems, we show the feasibility of achieving high-fidelity quantum logic gates.

II. SYSTEM AND MODEL

We start from the simple structure consisting of a NV ensemble and a LJJ. The NV ensemble is realized by NV centers with number N . Each NV center consists of a nitrogen impurity in the diamond lattice with a vacancy on a neighbouring lattice site. The ground state of a NV center has a spin one, with the sublevels $m_s = 0$ and $m_s = \pm 1$ separated by zero-field splitting D_{gs} . The NV center can be described by the Hamiltonian [21, 22]

$$H_{NV} = D_{gs}S_z^2 + E(S_x^2 - S_y^2) + g_e\mu_B\mathbf{B} \cdot \mathbf{S}, \quad (1)$$

where E is the strain-induced splitting coefficient, S_x , S_y , and S_z are the components of \mathbf{S} which denote the Pauli spin-one operators, $g_e = 2$ is the NV Landé factor, $\mu_B = 14\text{MHz mT}^{-1}$ is the Bohr magneton, and \mathbf{B} is the applied magnetic field. In this paper, the quantum information is encoded in sublevels $|m_s = 0\rangle \equiv |a\rangle$ and $|m_s = \pm 1\rangle \equiv |b\rangle$ serving as two logic states of a qubit. For a NV ensemble is composed of NV centers with number N , the ground state of a NV ensemble is defined as $|0\rangle = |a_1 a_2 \dots a_N\rangle$ while the excited state is defined as $|1\rangle = \sigma^+|0\rangle = (1/\sqrt{N})\sum_{k=1}^N |a_1 \dots b_k \dots a_N\rangle$ (all spins are in the ground state except the k -th spin) with operator $\sigma^+ = (\sigma^-)^\dagger = (1/\sqrt{N})\sum_{k=1}^N |b\rangle_k \langle a|$. The operator σ^+ can create symmetric Dicke excitation states. Thus, the Hamiltonian describing a NV ensemble reads [23]

$$H_{NVE} = \frac{\omega_{10}}{2}\sigma_z, \quad (2)$$

where ω_{10} is the energy difference between the lowest two levels with $|0\rangle$ and $|1\rangle$, and the operator $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ expresses the collective spin operator for the NV ensemble.

The quantum characters of LJJ have been widely studied in the early years [24, 25]. The idea of using a LJJ coupled to two charge qubits was first proposed in Ref. [26]. Then, generation of entanglement of two charge-phase qubits through a LJJ was discussed [27]. Also, generation of macroscopic entangled coherent states with LJJs was proposed [28]. The Hamiltonian of the LJJ can be written as [30] $H_J = E_C N^2 - E_J \cos \gamma$, where E_C expresses the charging

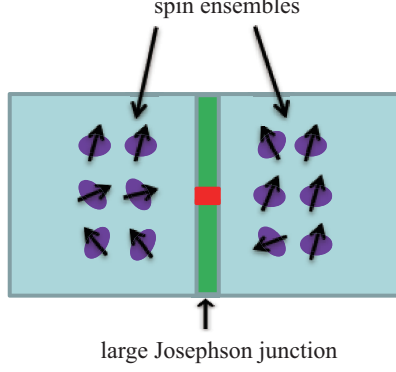


FIG. 1: (Color online) Schematic diagram of the proposed hybrid quantum system, which consists of a large Josephson junction and two spin ensembles.

energy, N is the excess Cooper pairs, E_J denotes the Josephson energy, and γ defines the phase drop across the LJJ. When the LJJ works in the phase regime, one can use a harmonic oscillator model to characterize the LJJ. Thus, the Hamiltonian for the LJJ is [30]

$$H_J = \omega a^\dagger a, \quad (3)$$

with bosonic operators $a^\dagger = \frac{\xi}{2}\gamma - i\frac{1}{2\xi}N$ and $a = \frac{\xi}{2}\gamma + i\frac{1}{2\xi}N$; and plasma frequency $\omega = \sqrt{8E_CE_J}$. Here, $\xi = (E_J/E_C)^{1/4}$.

The magnetic coupling strength between the k -th NV center and the LJJ can be roughly estimated as $g_k = 2g_e\mu_B B$. Here, B denotes magnetic field which can be estimated using the Biot-Savart law $B \simeq \mu_0 I_0 / 4\pi r$, where $\mu_0 = 4\pi \times 10^{-7} \text{ Nm}^2$, r expresses the distance between the LJJ and a NV center, and I_0 is critical current. The interaction Hamiltonian for a LJJ coupled to a NV ensemble can be represented by $\sum_k g_k (a^\dagger |a\rangle_{k,k} \langle b| + a |b\rangle_{k,k} \langle a|)$. In our architecture (Fig.1), NV ensembles are on the left and right of the LJJ, respectively. Here, we neglect the direct interaction of the two NV ensembles. Thus, the interaction Hamiltonian of two NV ensembles coupled to a LJJ can be written as

$$H_{int} = \sum_{j=1}^2 G_j a \sigma_j^+ + \text{H. c.} \quad (4)$$

where G_j is the collective coupling constant. Below, we give an estimation on this coupling strength $G_j = \sqrt{N}g_k$. According to experiments [25], the critical current I_0 can be chosen as $\sim 21\mu\text{A}$. The distance r from the center of the ensemble to a superconducting qubit is $\sim 1.2\mu\text{m}$ [12]. Given these parameters, we can estimate the coupling strength as $g_k \sim 0.62\text{MHz}$. The number N of NV centers takes $\sim 10^6$, the collective coupling constant is $G_j \sim 620\text{MHz}$. The decoherence rate of the NV ensemble $\Gamma \sim 1\text{MHz}$ [30] and the decay rate of the LJJ $\kappa \sim 3.3\text{MHz}$ [25] have been reported. Thus, this coupling is in the strong coupling regime.

III. GENERATION OF ENTANGLEMENT

We assume the system work within the large detuning condition $\delta \gg G_i$, where $\delta = \omega - \omega_{10}$ is the detuning between the LJJ and the NV ensemble. There is no energy exchange between the LJJ and each NV ensemble. The indirect interaction of the two NV ensembles can be induced by a LJJ without excitation. If we assume the LJJ is initially in the vacuum state, the effective Hamiltonian is given by

$$H_e = \sum_{j=1,2} \frac{G_j^2}{\delta} |1\rangle_{j,j} \langle 1| + \frac{G_1 G_2}{\delta} (\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+). \quad (5)$$

The first term describes the energy levels shifts, the last term describes the coupling of the two separated NV ensembles. In Ref. [31], a Hamiltonian similar to Eq. (5) was proposed using a cavity coupled to two atoms. We now consider the spontaneous emission of the NV ensembles. Under the assumption of weak decay of the LJJ, the evolution of the

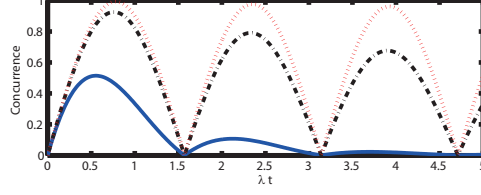


FIG. 2: (Color online) The time evolution of the concurrence of the state $|\Psi(t)\rangle$ for different spontaneous rates. Solid-blue lines, dot-black lines, and dash-red lines correspond to $\Gamma = \lambda$, $\Gamma = 0.1\lambda$, and $\Gamma = 0.01\lambda$, respectively.

system is governed by the conditional Hamiltonian

$$H_c = H_e - i \frac{\Gamma}{2} \sum_{j=1,2} |1\rangle_{j,j} \langle 1|. \quad (6)$$

Suppose the system is initially prepared in the state $|0\rangle_1|1\rangle_2$ and the coupling strength G_j is equal, i.e. $G_1 = G_2 = G$, then the state evolution of the system is given by

$$|\Psi(t)\rangle = C_1(t)|0\rangle_1|1\rangle_2 + C_2(t)|1\rangle_1|0\rangle_2, \quad (7)$$

with the coefficients $C_1(t) = \frac{1}{2} \exp(-\Gamma t/2)[1 + \exp(-i2\lambda t)]$ and $C_2(t) = \frac{1}{2} \exp(-\Gamma t/2)[\exp(-i2\lambda t) - 1]$ with the parameter $\lambda = G^2/\delta$, which denotes the effective coupling strength of two NV ensembles. The parameter λ can be estimated as 34.6MHz, which shows that in the large-detuning regime, the strong coupling between two NV ensembles, mediated by a LJJ, can be obtained. Eq. (7) expresses the entangled state of two NV ensembles. For different spontaneous rates Γ , we plot the concurrence change with λt in Fig.2. Obviously, concurrence can reach the maximum at some moment with a low spontaneous rate. In other words, the LJJ can induce maximal entanglement of two NV ensembles.

IV. REALIZING SINGLE-QUBIT LOGIC GATES

Recently, holonomic quantum computation based on nonadiabatic non-Abelian geometric phases has been proposed [32]. These kinds of geometric gates have been investigated in different physical systems [33–38]. We can construct the universal set of non-adiabatic holonomic single-qubit quantum gates based on Hamiltonian (4). We assume that the system is initially in the state $|100\rangle \equiv |1\rangle_1 \otimes |0\rangle_L \otimes |0\rangle_2$, i.e., they denote the states of the left NV ensemble, the LJJ and the right NV ensemble, respectively. Under the Hamiltonian (4), the state of the system evolves in the subspace $\{|\varphi_1\rangle = |100\rangle, |\varphi_2\rangle = |010\rangle, |\varphi_3\rangle = |001\rangle\}$. In such a subspace the eigenstates of the system are

$$|\psi_d\rangle = \frac{1}{\mathcal{G}}(-G_1|\varphi_1\rangle + G_2|\varphi_3\rangle), \quad (8)$$

$$|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}} \left[\frac{1}{\mathcal{G}}(G_2|\varphi_1\rangle + G_1|\varphi_3\rangle) \pm |\varphi_2\rangle \right], \quad (9)$$

with the corresponding eigenenergies given by $E_d = 0$ and $E_{\pm} = \pm \sqrt{G_1^2 + G_2^2} = \mathcal{G}$. $|\psi_d\rangle$ expresses the dark state, which is decoupled from the Hamiltonian and undergoes no transition during the application of the driving fields. The bright state is $|\psi_b\rangle = \frac{1}{\mathcal{G}}(G_2^*|\varphi_1\rangle + G_1^*|\varphi_3\rangle)$. It is obvious that the effective Rabi frequency between the bright state

$|\psi_b\rangle$ and the quantum state $|\varphi_2\rangle$ is \mathcal{G} . Without loss of generality, we set $G_1/\mathcal{G} = \cos(\theta/2)$ and $G_2/\mathcal{G} = e^{i\phi} \sin(\theta/2)$, with $0 \leq \theta \leq \pi$ decided by the ratio between the coupling strength G_1 and G_2 , and $0 \leq \phi \leq 2\pi$ depending on the relative phase. When condition $\int_0^T \mathcal{G} dt = \pi$ is satisfied, the dark state and bright state undergo a cyclic evolution. In the computational basis $\{|\varphi_1\rangle, |\varphi_3\rangle\}$, the final evolution operator is

$$\begin{pmatrix} -\cos\theta & \sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & \cos\theta \end{pmatrix}, \quad (10)$$

which can be used to realize any single-qubit rotation.

V. REALIZING TWO-QUBIT LOGIC GATES

Two-qubit logic gates play a key role in quantum computation and quantum information. A multiqubit gate can be composed of single-qubit and two-qubit gates. In this section, we discuss how to realize a two-qubit phase gate and a two-qubit π phase-swap gate with two spin ensembles, respectively.

A. phase gate

In order to implement a phase gate, an external signal is applied to the LJJ via an on-chip antenna. The Hamiltonian of the external signal driving the LJJ can be modeled by [39]

$$H_d = \varepsilon(a^\dagger e^{-i\omega_d t} + a e^{i\omega_d t}), \quad (11)$$

where parameters ε and ω_d express the amplitude and the frequency of the external signal, respectively. Then, in the Schrödinger picture, the total Hamiltonian of the system can be written as

$$\begin{aligned} H_t &= H_J + H_{NVE} + H_{int} + H_d \\ &= \omega a^\dagger a + \frac{\omega_{10}}{2} \sigma_z + \sum_{j=1}^2 G_j (a \sigma_j^+ + a^\dagger \sigma_j^-) \\ &\quad + \varepsilon(a^\dagger e^{-i\omega_d t} + a e^{i\omega_d t}). \end{aligned} \quad (12)$$

We introduce a displacement-transformation operator $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$, where α is a complex number. After the displacement-transformation $D(\alpha)$ of the Hamiltonian (9), we obtain a new Hamiltonian $H_T = D^\dagger(\alpha) H_t D(\alpha) - i D^\dagger(\alpha) \dot{D}(\alpha)$, where the expression of complex number is $\dot{\alpha} = -i\omega\alpha - i\varepsilon e^{-i\omega_d t}$. The drive amplitude ε is independent of time. A rotating frame transformation $U_R = \exp[-i\omega_d(\sigma_z^j + a^\dagger a)t]$ is applied to Hamiltonian H_T , then in the interaction picture, we take $\omega_{10} = \omega_d$, then the H_T becomes [40]

$$H_T = \sum_{j=1,2} \left[\Omega_j \sigma_x^j + G_j (\sigma_+^j a e^{-i\delta t} + \sigma_-^j a^\dagger e^{i\delta t}) \right], \quad (13)$$

where $\Omega_j = \varepsilon G_j / \delta$. The Hamiltonian (10) can be divided two parts, which include the free term $H'_0 = \sum_{j=1,2} \Omega_j \sigma_x^j$ and the interaction term $H'_I = \sum_{j=1,2} G_j (\sigma_+^j a e^{-i\delta t} + \sigma_-^j a^\dagger e^{i\delta t})$. In the interaction picture, we define the new orthogonal bases $|\pm\rangle_j = (|0\rangle_j \pm |1\rangle_j) / \sqrt{2}$. Under the strong driving regime i.e. $\Omega_j \gg \{\delta, G_j\}$, we eliminate the fast-oscillating terms and obtain effective Hamiltonian [41]

$$H_{eff} = \sum_{j=1,2} \frac{G_j}{2} \sigma_x^j (a^\dagger e^{-i\delta t} + a e^{i\delta t}). \quad (14)$$

If the evolution time satisfies $t = \tau_n = 2n\pi/\delta$ for integer n , the direct interaction between the NV ensembles and a LJJ can be eliminated. Here, if we neglect a trivial universal phase factor, the evolution operator of Eq. (14) can be expressed as $U(n) = \exp[-iB(n)\sigma_x^1\sigma_x^2]$ with the parameter $B(n) = -n\pi G_1 G_2 / \delta^2$. Returning to the Schrödinger

picture, the time evolution operator is given by

$$\begin{aligned} U(\tau) &= e^{-iH'_0\tau} e^{-iB(n)\sigma_x^1\sigma_x^2}, \\ &= e^{-i\Omega_1\tau\sigma_x^1} e^{-i\Omega_2\tau\sigma_x^2} e^{-iB(n)\sigma_x^1\sigma_x^2}. \end{aligned} \quad (15)$$

If the conditions $\Omega_1\tau = \Omega_2\tau = -B(n) = \theta$ are satisfied by controlling frequencies Ω_1 and Ω_2 , the total time evolution operator can be rewritten as

$$U(\theta) = \exp[-i\theta(\sigma_x^1 + \sigma_x^2 - \sigma_x^1\sigma_x^2)]. \quad (16)$$

Here, we choose the following bases $\{|+\rangle_1|+\rangle_2, |+\rangle_1|-\rangle_2, |-\rangle_1|+\rangle_2, |-\rangle_1|-\rangle_2\}$, and set $4\theta = (2m+1)\pi$ (where m is an integer). A two-qubit phase gate is realized [27, 42]

$$U_{CP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (17)$$

According to the above parameters experimental value, we estimate the time of implementing two-qubit phase gate. The plasma frequency the LJJ $\omega = 2\pi \times 6.9\text{GHz}$ [25] and the frequency of the NV ensemble $\omega_{10} = 2\pi \times 2.87\text{GHz}$ [23] have been reported, respectively. When we take $n = 1$, the time of achieving phase gate is $\tau_{cp} \approx 0.41\text{ns}$.

B. π phase-swap gate

Here, we choose appropriate conditions as follows: (i) two NV ensembles equally couple to the LJJ, i.e. $G_1 = G_2 = G$; (ii) evolution time takes $t = \tau_k = \pi\delta/2G^2$. In the basis of two NV ensembles $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$, the matrix of the evolution operator of the Hamiltonian (5) reads

$$U_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (18)$$

where neglect a global phase factor. This quantum logic gate can be equivalent to a π -phase gate and a swap gate. If it is realized experimentally in the future, this logic gate could be useful for quantum and information and quantum computation. Next, we estimate the shortest time $\tau_k \sim 103.11\text{ ns}$ for realizing this π phase-swap gate.

VI. DISCUSSION AND CONCLUSION

The recent experiments have reported: the electron spin relaxation time T_1 of the NV center is 28 s at low temperature [43], the dephasing time T_2 of isotopically pure diamond sample is 2 ms [44], the NV ensemble coherence time T approach 1 s [45]. The time τ_{cp} and τ_k are much shorter than the T_1 , T_2 and T . It is necessary to investigate the influence of decoherence of the system on the quantum logic gates. If two NV ensembles have identical relaxation rate Γ_1 and dephasing rate Γ_2 , the dynamics of the lossy system is determined by the following master equation

$$\begin{aligned} \dot{\rho} &= -i[H_{eff}, \rho] + \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ &+ \frac{\Gamma_1}{2} \sum_{j=1,2} (\sigma_z^j \rho \sigma_z^j - \rho) \\ &+ \frac{\Gamma_2}{2} \sum_{j=1,2} (2\sigma_j^- \rho \sigma_j^+ - \rho \sigma_j^+ \sigma_j^- - \sigma_j^+ \sigma_j^- \rho), \end{aligned} \quad (19)$$

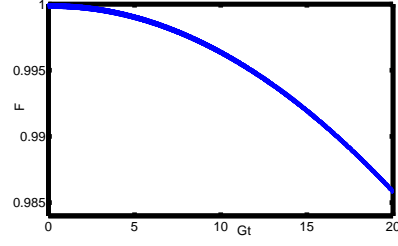


FIG. 3: (Color online) The fidelity of the phase gate U_{CP} versus the evolution time Gt .

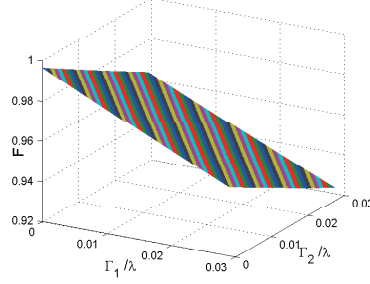


FIG. 4: (Color online) The fidelity of the π phase-swap gate U_s versus the decay rates Γ_1 and Γ_2 .

for realizing the phase gate. Here, κ is the decay rate of the LJJ. For achieving the π phase-swap gate, the master equation is given by

$$\begin{aligned} \frac{d\rho}{dt} = & -i[H_e, \rho] + \frac{\Gamma_1}{2} \sum_{j=1,2} (\sigma_z^j \rho \sigma_z^j - \rho) \\ & + \frac{\Gamma_2}{2} \sum_{j=1,2} (2\sigma_j^- \rho \sigma_j^+ - \rho \sigma_j^+ \sigma_j^- - \sigma_j^+ \sigma_j^- \rho). \end{aligned} \quad (20)$$

The fidelity is defined as [46] $F = \overline{\langle \Psi | U^\dagger \rho_t U | \Psi \rangle}$, where the overline indicates average overall possible initial states $|\Psi\rangle$, U is the ideal two-qubit operation, and ρ_t is the final density operator after the U operation performed in a real situation. In Fig. 3, we plot fidelity of the phase gate with Gt for the decay rate $\kappa = \Gamma_1 = \Gamma_2 = 1\text{MHz}$. We plot the fidelity F of the π phase-swap gate with the dephasing rate Γ_1 and relaxation rate Γ_2 in Fig.4. Obviously, the high-fidelity quantum logic gates can be realized when we use the previously reported decay rate $\Gamma \sim 1\text{MHz}$.

Comparing with the coupling between the NV ensembles and the superconducting flux qubits [11, 12, 20], our proposal is simpler in experiment. First, the preparation technology of the LJJ is much easier than that of the flux qubits. Second, the quantum information is encoded in sublevels $|m_s = 0\rangle \equiv |a\rangle$ and $|m_s = \pm 1\rangle \equiv |b\rangle$ serving as two logic states of a qubit, thus, there is no need of an external magnetic field to remove the degeneracy of spin sublevels $|m_s = \pm 1\rangle$.

In previous works on NV ensembles, the fidelity of the phase gate is about 98.23% within an operation time $\sim 93.87\text{ns}$ [47]. However, in our proposal, the gate operation time is $\sim 0.41\text{ns}$ and the gate can be implemented with a high-fidelity greater than 99%.

In summary, we have proposed a strong-coupling hybrid quantum system. With this system, the entangled state,

non-adiabatic holonomic single-qubit gates, the two-qubit π phase-swap gate and phase gate can be implemented. With the realistic experimental parameters, we showed that the gate operation time is much shorter than the decoherence time of the system. Hence, our scheme is implementable with the current experimental technology.

Acknowledgments

FYZ was supported by the National Science Foundation of China under Grant Nos. [11505024, 11447135 and 11505023], and the Fundamental Research Funds for the Central Universities No. DC201502080407. CPY was supported in part by the National Natural Science Foundation of China under Grant Nos. 11074062 and 11374083, and the Zhejiang Natural Science Foundation under Grant No. LZ13A040002. This work was also supported by the funds from Hangzhou City for the Hangzhou-City Quantum information and Quantum Optics Innovation Research Team. And we acknowledge useful discussions with Zhang-qi Yin.

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